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INFLUENCE OF THE PLASTIC POTENTIAL ON THE MECHANICAL RESPONSE OF THERMOPLASTIC COMPONENTS

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Summary. This paper presents the influence of the plastic potential in the mechanical response of thermoplastic components. This study is based on a recently hyperelastic-viscoplastic constitutive model developed for polymeric materials. Assuming a non-associative plasticity framework different plastic potentials are considered in this study (e.g. isochoric, quasi-linear, parabolic and elliptic). The present model is intended to be used to characterize closer the matrix behavior of polymeric based composite materials under a micro-mechanics framework.

1 THE MBR CONSTITUTIVE MODEL

Polymers are increasingly being used in the transport industry, specially, in structural components related to passengers or pedestrian safety. In this direction, a new hyperelastic-viscoplastic constitutive model for thermoplastics (under isothermal conditions) has been developed by Polanco-Loria et al.[1] (see Fig. 1). The model is a physically-based constitutive model, involving the typical mechanisms of the elastic behavior of polymers, i.e. relative rotation around backbone carbon-carbon bonds and entropy change by un-coiling molecule chains. In addition, viscoplastic flow associated with relative movement between molecules is included. Historically, the development of this model goes back to the work by Haward and Thackray[2] and further developed by Boyce[3] and Boyce et al.[4], who assumed that the total stress was the sum of an inter-molecular and intra-molecular contribution denoted Part A and Part B, respectively.

Shortly, the elastic response of part A is described by a compressible Neo-Hookean material where the Cauchy stress tensor reads:

$$\boldsymbol{\sigma}_A = \frac{1}{J_A^e}\left[\lambda \ln J_A^e\, \mathbf{I} + \mu(\mathbf{B}_A^e - \mathbf{I})\right] \tag{1}$$

In addition, yield condition assumes a pressure-sensitivity criterion based on the work of Raghava et al.[5]

$$\bar{\sigma}_A = \frac{(\alpha-1)I_{1A} + \sqrt{(\alpha-1)^2 I_{1A}^2 + 12\alpha J_{2A}}}{2\alpha} \tag{2}$$

In order to control the plastic dilatation, a non-associative flow rule is introduced where a Raghava-like plastic potential is defined as

$$g_A = \frac{(\beta-1)I_{1A} + \sqrt{(\beta-1)^2 I_{1A}^2 + 12\beta J_{2A}}}{2\beta} \geq 0 \tag{3}$$

With respect to the plasticity response of Part A (see Fig.1) the model was enhanced to include isotropic hardening/softening behavior according to Voce's saturation model[6]:

$$R = \left(\sigma^{sat} - \sigma_T\right)\left[1 - \exp^{-H\bar{\varepsilon}^p}\right] \tag{4}$$

where, R is the stress hardening level. The hardening/softening modulus is represented by H, the saturation and yield tensile stress by σ^{sat} and σ_T, respectively. Hence, for the hardening case $\sigma^{sat} > \sigma_T$ while for the softening case $\sigma^{sat} < \sigma_T$.

The part B includes the deformation gradient $\mathbf{F}_B = \mathbf{F}_A = \mathbf{F}$, representing the network orientation. The network resistance is assumed to be hyperelastic. The Cauchy stress-stretch relation is used as the original definition of Boyce et al. [4] :

$$\boldsymbol{\sigma}_B = \frac{1}{J}\left[\frac{C_R}{3}\frac{\sqrt{N}}{\bar{\lambda}}\mathcal{L}^{-1}\left(\frac{\bar{\lambda}}{\sqrt{N}}\right)(\mathbf{B}_B^* - \bar{\lambda}^2\mathbf{I})\right] \tag{5}$$

The constitutive model requires 11 parameters to be identified:

- **Spring A** represents the initial elastic stiffness with a Neo-Hookean formulation. There are two elastic coefficients E (Young's modulus) and ν (Poisson's ratio).
- **Friction element A** models the yielding process with pressure dependency and a non-associative flow rule. Three parameters in this friction element are needed: the uniaxial yield tensile stress σ_T, the pressure sensitive parameter α and the volumetric

plastic strain control parameter β. The hardening/softening behaviour necessitates two additional terms (see Eqn. 1): H and σ^{sat}.

- **Dashpot A** is included to represent the rate dependence of the material. The visco-plastic multiplier uses a linear (log scale) strain rate law characterized by two parameters: the reference strain rate $\dot{\varepsilon}_0$ and the strain rate coefficient C.
- **Spring B** represents the elongation of the molecule chains, here modeled with a hyperelastic law. Only the distortional stress-stretch relation is used here where two hardening coefficients C_R and N need to be identified.

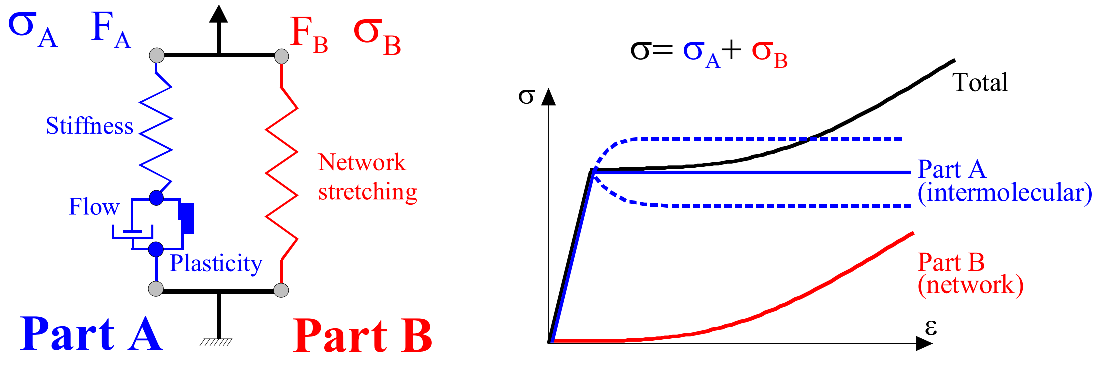

Figure 1: Constitutive model with inter-molecular (A) and network (B) contributions

A complete description of the parameter identification process has been proposed elsewhere[7]. The model will be referred here as the modified Boyce-Raghava (MBR) model.

2 INFLUENCE OF THE PLASTIC POTENTIAL

The original work[1] proposes a non-associative plastic potential (see Eqn. 3) to handle the volumetric plastic flow, commonly observed in polymers. The apparently drawback of such proposal is the dilation behavior under compressive stresses. For this reason, a closer study on the importance of such plastic potential is considered here. In addition to the classical isochoric assumption three different potentials, all of them giving the *same volumetric* plastic contribution, have been considered: quasi-linear, parabolic (Eqn. 3) and elliptic. All of them can be calibrated to give the same plastic volumetric response in uniaxial tension. Qualitatively the quasi-linear model predicts less volumetric plastic strain than the parabolic and elliptic in the high triaxial state of stress. Only the elliptic model is capable of predicting compaction for negative pressures. This model however, requires an additional parameter. An illustration of the plastic potentials studied is indicated in Figure 2.

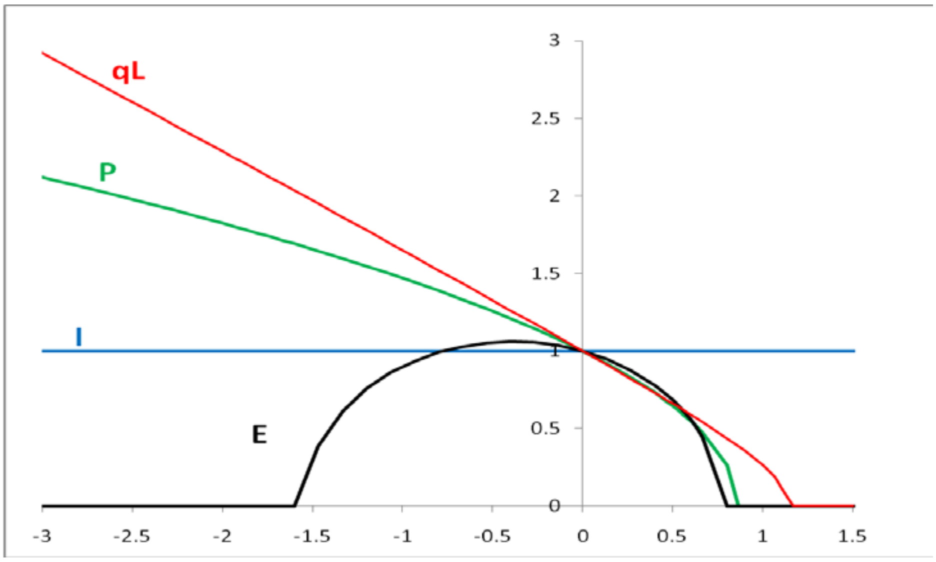

Figure 2: Different plastic potentials considered

3 EXPERIMENTAL DATA USING A PP COPOLYMER MATERIAL

3.1 Introduction

The experimental results of a commercial impact-modified PP used for injection molded automotive exterior parts are used for illustration purposes[7]. This PP compound is a 20 % mineral filled and rubber modified. A complete description of the parameter identification process was proposed by Polanco-Loria et al.[7] and the predictions of the constitutive model (assuming the original parabolic law) are presented in Figure 3. The material parameters assumed for the PP copolymer are indicated in Table 1.

-**Table 1**: Material parametrs for the PP copolymer

E MPa	ν	C	$\dot{\varepsilon}_0$ 1/s	σ_T MPa	σ_{Sat} MPa	H MPa	C_R MPa	N	β	α
1500	0.4	0.08	2x10^{-4}	14.0	11.5	8.0	1.60	5.0	1.47	1.17

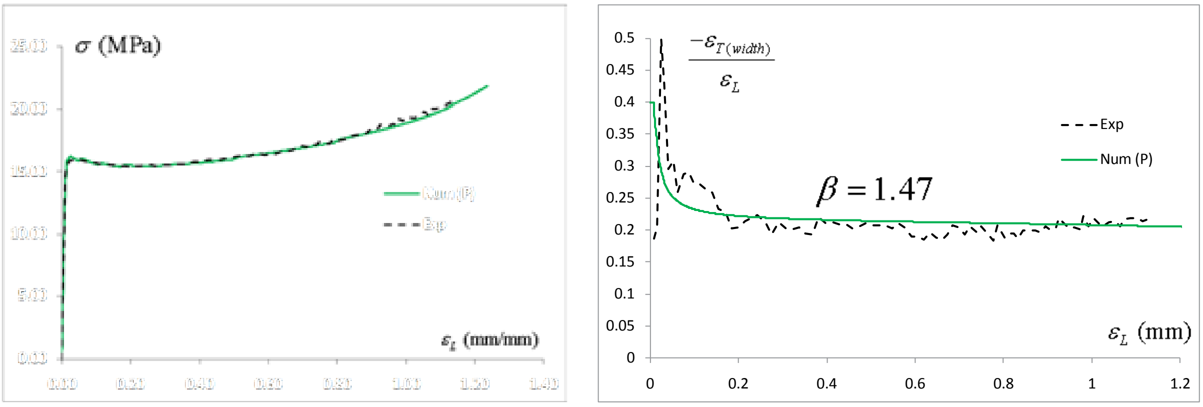

Figure 3: Uniaxial tensile stress-strain response and prediction of the "Poisson" ratio variation

3.2 Material calibration for the quasi-linear, elliptic and isochoric plastic flow rules

The material identification procedure was applied to the quasi-linear, elliptic and isochoric assumptions based on the experimental tension test results of Fig. 3. As one can expect, the stress-strain response of these three models are similar, as illustrated in Fig 4. In this figure we included the numerical and experimental response of the isochoric model. The differences between these responses clearly indicates damage activity in form of void grow and crazes formation.

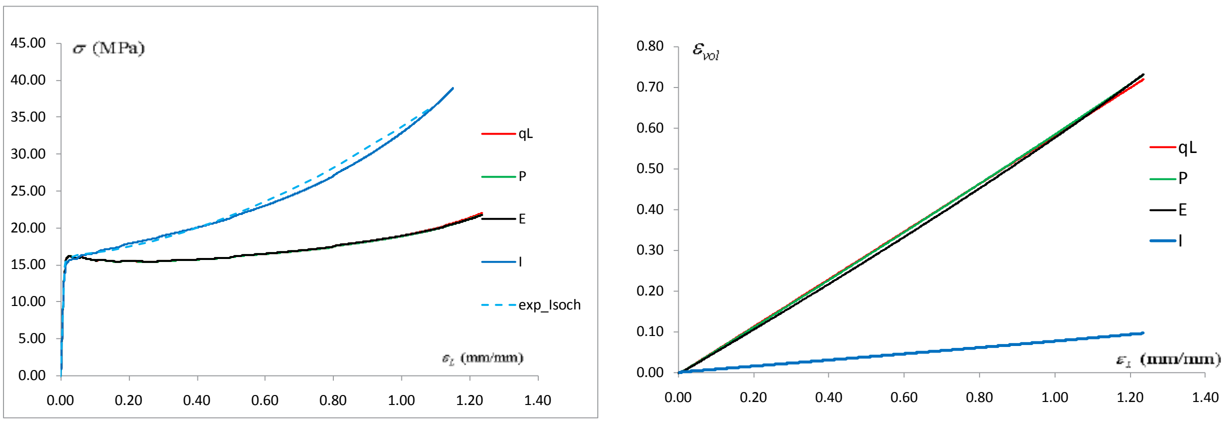

Figure 4: Uniaxial tensile stress-strain response and prediction of the volumetric strains

The total volumetric strain response predicted by the models is also indicated in figure 4. Once again, the volumetric response of the parabolic, quasi-linear and elliptic is similar. Large differences in the volumetric strain response between these models and the isochoric one (e.g. only predicts elastic strains) are observed (bleu line).

Now, a more reliable comparison can be performed to assess the influence of the plastic potential in the mechanical behavior of thermoplastic components.

4 CONCLUSIONS

- The study of the plastic potential on the mechanical response of thermoplastic components has been proposed.
- In addition to the isochoric assumption three potentials were considered: quasi-linear, parabolic and elliptic. With proper calibration all of them give the same response under uniaxial tension loading.
- Several examples of thermoplastic components will be given at the oral presentation (e.g. beam and plates)
- The present model is part of a new development to characterize closer the matrix behavior of polymeric based composite materials under a micro-mechanics framework.

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